A Framework for Qualitative Multi-Criteria Preferences: Extended Abstract*

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Introduction A key challenge in the representation of qualitative, multi-criteria preferences is to find a compact and expressive representation. Various frameworks have been introduced, each of which with its own distinguishing features. In this paper we introduce a new representation framework called qualitative preference systems (QPS), which combines priority, cardinality and conditional preferences. Moreover, the framework incorporates knowledge that serves two purposes: to impose (hard) constraints, but also to define new (abstract) concepts.

QPSs are based on the lexicographic rule studied in [1]. This rule is a fundamental part of the framework presented as it offers a principled tool for combining basic preferences. We believe this ability to combine preferences is essential for any practical approach to representing qualitative preferences. It is needed in particular for constructing multi-criteria preferences. It is not sufficient, however, since more expressivity is needed and useful in practice. Therefore, QPSs in addition provide a tool for representing knowledge, for abstraction, for counting, and provide a layered structure for representing preference orderings. QPSs are able to represent various strategies for defining preference orderings, and are able to handle conditional preferences. Logical Preference Description language (LPD; [3]) can be embedded into the QPS framework and that there is an order preserving embedding of CP-nets [2] in the QPS framework. These embeddings provide a representation that is just as succinct as the LPD expressions and CP-nets.

Qualitative Preference Systems The main aim of a QPS is to determine preferences between *outcomes* in a purely qualitative way. An outcome is an assignment of values to a set of relevant variables. Every variable has its own domain of possible values. Constraints on the assignments of values to variables are expressed in a knowledge base. Outcomes are defined as variable assignments that respect the constraints in the knowledge base. The preferences between outcomes are based on multiple *criteria*. Every criterion can be seen as a *reason* for preference, or as a preference from one particular *perspective*. We distinguish between simple criteria that are based on a single variable and compound criteria that combine multiple criteria in order to determine an overall preference. There are two kinds of compound criteria: lexicographic criteria and cardinality criteria.

Definition 1. (Qualitative preference system) A *qualitative preference system (QPS)* is a tuple $\langle Var, Dom, K, \mathcal{C}\rangle$. *Var* is a finite set of *variables*. Every variable $X \in Var$ has a domain $Dom(X)$ of possible values. K (a *knowledge base*) is a set of constraints on the assignments of values to the variables in *Var*. A *constraint* is an equation of the form $X = Expr$ where $X \in Var$ is a variable and *Expr* is an algebraic expression that maps to $Dom(X)$. An *outcome* α is an assignment of a value $x \in Dom(X)$ to every variable $X \in Var$, such that no constraints in K are violated. α_X denotes the value of variable X in outcome α. $\mathcal{C}$ is a finite rooted tree of criteria, where leaf nodes are simple criteria and other nodes are compound criteria. Child nodes of a compound criterion are called its subcriteria. Weak preference between outcomes by a criterion c is denoted by the relation $\succeq_c$. $\succ_c$ denotes the strict subrelation, $\approx_c$ the indifference subrelation.

Simple criteria A simple criterion specifies a preference ordering on the values of a single variable. Its preference between outcomes is based solely on the value of this variable in the considered outcomes.

*This is an abstract of [4]. More information about the ideas in this abstract and references to relevant literature can be found there.

Definition 2. (Simple criterion) A *simple criterion* c is a tuple $\langle X_c, \trianglerighteq_c \rangle$, where $X_c \in Var$ is a variable, and $\trianglerighteq_c$, a preference relation on the possible values of X_c, is a preorder on $Dom(X_c)$. $\triangleright_c$ is the strict subrelation, $\doteq_c$ is the indifference subrelation. We call c a *Boolean simple criterion* if X_c is Boolean and $\top \triangleright_c \bot$. A simple criterion $c = \langle X_c, \trianglerighteq_c \rangle$ *weakly prefers* an outcome α over an outcome β, denoted $\alpha \succeq_c \beta$, iff $\alpha_{X_c} \trianglerighteq_c \beta_{X_c}$.

Observation 1. Let $c = \langle X_c, \trianglerighteq_c \rangle$ be a simple criterion. Then $\succeq_c$ is a preorder. If $\trianglerighteq_c$ is total, then so is $\succeq_c$.

Lexicographic criteria A lexicographic criterion consists of a set of subcriteria and an associated priority order (a strict partial order, which means that no two subcriteria can have the same priority). It weakly prefers outcome α over outcome β if for every subcriterion, either this subcriterion weakly prefers α over β, or there is another subcriterion with a higher priority that strictly prefers α over β. This definition of preference by a lexicographic criterion is equivalent to the priority operator as defined by [1]. It generalizes the familiar rule used for alphabetic ordering of words, such that the priority can be any partial order and the combined preference relations can be any preorder.

Definition 3. (Lexicographic criterion) A *lexicographic criterion* c is a tuple $\langle C_c, \rhd_c \rangle$, where C_c is a nonempty set of criteria (the *subcriteria* of c) and $\rhd_c$, a *priority relation* among subcriteria, is a strict partial order (a transitive and asymmetric relation) on C_c. A lexicographic criterion $c = \langle C_c, \rhd_c \rangle$ *weakly prefers* an outcome α over an outcome β, denoted $\alpha \succeq_c \beta$, iff $\forall s \in C_c (\alpha \succeq_s \beta \vee \exists s' \in C_c (\alpha \succ_{s'} \beta \wedge s' \rhd_c s))$.

Proposition 1. Let $c = \langle C_c, \rhd_c \rangle$ be a lexicographic criterion. If for all subcriteria $s \in C_c$, $\succeq_s$ is a preorder, then the relation $\succeq_c$ is also a preorder.

Cardinality criteria Like a lexicographic criterion, a cardinality criterion combines multiple criteria into one preference ordering. Unlike a lexicographic criterion, priority between subcriteria is not a strict partial order, but all subcriteria have the same priority. A cardinality criterion weakly prefers an outcome α over an outcome β if it has at least as many subcriteria that strictly prefer α over β as criteria that do not weakly prefer α over β.

Definition 4. (Cardinality criterion) A *cardinality criterion* c is a tuple $\langle C_c \rangle$ where C_c is a nonempty set of criteria (the *subcriteria* of c). A cardinality criterion $c = \langle C_c \rangle$ *weakly prefers* an outcome α over an outcome β, denoted $\alpha \succeq_c \beta$, iff $|\{s \in C_c \mid \alpha \succ_s \beta\}| \geq |\{s \in C_c \mid \alpha \not\succeq_s \beta\}|$.

Proposition 2. Let $c = \langle C_c \rangle$ be a cardinality criterion such that for all $s \in C_c$, s is a Boolean simple criterion. Then $\succeq_c$ is a preorder.

[1] showed that the only operator to combine *any arbitrary* preference relations that satisfies the desired properties IBUT (independence of irrelevant alternatives, based on preferences only, unanimity with abstentions, and preservation of transitivity) is the priority operator, which assumes that priority is a partial order. We observe here that if only *Boolean* preference relations (such as those resulting from Boolean simple criteria) are combined, the cardinality-based rule, in which all combined relations have equal priority, also satisfies the properties IBUT. Requiring antisymmetry in this case would unneccessarily restrict the expressivity.

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