The Convergence of Reciprocation

(Extended Abstract)

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ABSTRACT

People often interact repeatedly: with relatives, through file sharing, in politics, etc. Many such interactions are reciprocal: reacting to the actions of the other. In order to facilitate decisions regarding reciprocal interactions, we analyze the development of reciprocation over time. To this end, we propose a model for such interactions that is simple enough to enable formal analysis, but is sufficient to predict how such interactions will evolve. Inspired by existing models of international interactions and arguments between spouses, we suggest a model with two reciprocating attitudes where an agent's action is a weighted combination of the others' last actions (reacting) and either i) her innate kindness, or ii) her own last action (inertia). We analyze a network of repeatedly interacting agents, each having one of these attitudes, and prove that their actions converge to specific limits. Convergence means that the interaction stabilizes, and the limits indicate the behavior after the stabilization. For two agents, we describe the interaction process and find the limit values. For a general connected network, we find these limit values if all the agents employ the second attitude, and show that the agents' actions then all become equal. In the other cases, we study the limit values using simulations. We discuss how these results predict the development of the interaction and constitute the first step towards helping agents decide on their behavior.

Keywords

reciprocal interaction; repeated reciprocation; behavior; convergence; Perron-Frobenius; agent's influence

1. INTRODUCTION

Interaction is central in life, e.g., at school, on the road, and in politics. We aim to facilitate decision support to people and applications that interact, such as a self-driving car. To this end, we need to predict interaction. Instead of being economically rational, people tend to adopt other ways of behavior [6]. Furthermore, people tend to reciprocate, i.e., react on the past actions of others [2]. Some extant work studies how reciprocation has emerged. For example, Axelrod [1] shows that (discrete) reciprocity is rational to egoists, while [3] reasons that people sometimes exhibit irrational reciprocity, and gave explanations of it. Reciprocity seems to be intrinsic [7]. On another avenue, given the reciprocal tendencies, several works analyze why they make interactions develop in certain ways. For instance, [2, 5] define a game where utility depends on reciprocal behavior. Since no analysis considers non-discrete lengthy interactions, caused by reciprocation, we ask how reciprocal interactions evolve with time. This will facilitate decision making by predicting what approach will benefit the agents more.

Consider an interaction network where vertices stand for agents N, and two agents interact if and only if they are neighbors. We model an action by a single number, which represents the value of that action to the recipient. This number is defined to be a convex combination between the inner self and outer influence. Every agent i has her kindness k_i, representing the default value of her action, and a direct and a social reciprocation coefficient, r_i and r'_i, respectively, representing her inclination to react to a given agent and to all the agents she interacts with, respectively. Denote the action of agent i on j at time t by $x_{i,j}(t)$, and the set of the neighboring agents of i by $\mathrm{N}(i)$. We assume that all the agents act on times $T = \{0, 1, \ldots\}$. We define two reciprocation attitudes. In the *fixed* attitude, the action of agent i on j at time t is defined by the kindness, the reaction to the other's action, and the reaction to the average of the actions of the neighbors:

$$(1 - r_i - r'_i) \cdot k_i + r_i x_{j,i}(t-1) + r'_i \frac{\sum_{j \in \mathrm{N}(i)} x_{j,i}(t-1)}{|\mathrm{N}(i)|},$$

In the *floating* attitude, the first term is own action, namely:

$$(1 - r_i - r'_i) \cdot x_{i,j}(t-1) + r_i x_{j,i}(t-1) + r'_i \frac{\sum_{j \in \mathrm{N}(i)} x_{j,i}(t-1)}{|\mathrm{N}(i)|}.$$

Defining action or state by a linear combination of the other side's actions and own actions and qualities is also used to analyze arms race [8] and spouses' interaction [4] (piecewise linear in this case). This model defines an infinite sequence of actions for every agent, and predicting this process would allow setting up efficient reciprocation.

2. PAIRWISE INTERACTION

When only a pair of agents interact, assume w.l.o.g. that $r'_1 = r'_2 = 0$. We prove convergence as time approaches infinity. Convergence means that the interaction stabilizes with time. When both agents are *fixed*, they converge in an oscillating manner (see Figure 1), and $\lim_{t\to\infty} x_{i,j}(t) =$

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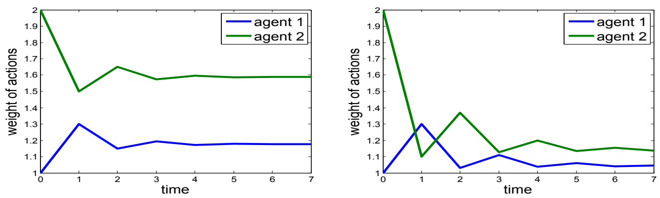

Figure 1: Simulation of actions for $r_1+r_2 < 1$, $r_2 = 0.5$ on the left, and $r_1+r_2 > 1$, $r_2 = 0.9$ on the right. It is a *fixed* - *fixed* reciprocation, with $k_1 = 1, k_2 = 2, r_1 = 0.3$.

$\frac{(1-r_i)k_i+r_i(1-r_j)k_j}{1-r_i r_j}$. When at least one agents is *floating*, they converge to a common limit. If both are *floating*, the limit is $\frac{r_2}{r_1+r_2}k_1 + \frac{r_1}{r_1+r_2}k_2$; if i is *fixed* and j is *floating*, the limit is k_i, and if $r_1 + r_2 \leq 1$, the convergence is monotonic from some moment on. The limits imply that if you consider your kindness while reciprocating (*fixed*), then, asymptotically, your actions values get closer to your kindness than if you consider it only at the outset. Thus, persistence makes the interaction go your way. Another interesting result is that always $k_i \leq k_j \Rightarrow \lim_{t\to\infty} x_{i,j}(t) \leq \lim_{t\to\infty} x_{j,i}(t)$.

3. MULTI-AGENT INTERACTION

For multiple agents, we employ the Perron-Frobenius theorem. When all r'_i are positive, we prove a geometrically fast convergence. We also prove that when at least one agent is *fixed*, every $\lim_{t\to\infty} x_{i,j}(t)$ is a positive combination of the kindnesses of the *fixed* agents. If all the *fixed* agents have the same kindness k, this is also the (common) limit. If all the agents are *floating*, we show that the actions of all the agents converge to a common limit, which is

$$\frac{\sum_{i\in N}\left(\frac{|\mathrm{N}(i)|}{r_i+r'_i}\cdot k_i\right)}{\sum_{i\in N}\left(\frac{|\mathrm{N}(i)|}{r_i+r'_i}\right)}. \quad (1)$$

The convergence partially explains personal styles of behavior. The commonality alludes to the formation of organizational (sub)cultures. The limit being a combination of the kindness values of certain agents means that the kindness of an agent has either no influence, or it constitutes a linear term. Observe that when all the agents are *floating*, the influence of an agent on the common limit is proportional to the number of agents on whom she may act, and inversely proportional to her tendency to reciprocate, which may be called stability. This explains that persistence makes an agent more influential on the actions in the interaction.

An interesting example is a regular interaction network, where $|\mathrm{N}(i)|$ is the same for all i. This holds, for instance, for cliques, modeling small groups of people or countries, and for cycles, modeling circular computer networks. In this case, Eq. (1) becomes $\frac{\sum_{i\in N}\left(\frac{k_i}{r_i+r'_i}\right)}{\sum_{i\in N}\left(\frac{1}{r_i+r'_i}\right)}$. Another interesting example is a star network, modeling a supervisor of separate entities. Assume w.l.o.g. that agent 1 is the center, and the common limit becomes $\frac{\frac{|N|-1}{r_1+r'_1}\cdot k_1+\sum_{i\in N\setminus\{1\}}\left(\frac{k_i}{r_i+r'_i}\right)}{\frac{|N|-1}{r_1+r'_1}+\sum_{i\in N\setminus\{1\}}\left(\frac{1}{r_i+r'_i}\right)}$.

We show that to maximize the limit value of the actions by setting her r_i (or r'_i), an agent needs an extreme value of her r_i (or r'_i). When all the agents are *floating*, Eq. (1) implies that the dependency of the limits on the extent of reciprocativeness r_i (or on r'_i) is monotonic, so maximizing the limits requires being either completely reciprocal or not at all. To study this when we do not know the limit, we simulate the interaction, obtaining plots like those in Figure 2. In all the considered cases, this dependency is monotonic. For the dependency on r'_i, the obtained graphs are also monotonic.

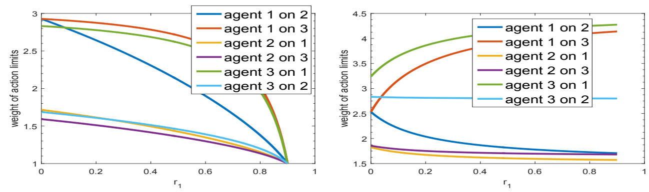

Figure 2: The simulated limits of actions as functions of r_1, for $r_2 = 0.1, r_3 = 0.6, r'_1 = 0.1, r'_2 = 0.4, r'_3 = 0.1$, $k_1 = 3, k_2 = 1, k_3 = 5$. On the left, agent 1 and 2 are the only *fixed* agents, while on the right, 1 is the only *floating* agent.

To advise more constructively about what agents' parameters and attitude are useful, defining utilities and considering choosing one's parameters, so as to maximize own utility is very promising for future work. To summarize, we analyze the interaction process, in order to predict reciprocal interaction and thereby, to facilitate decisions regarding how to reciprocate.

The full version, named "Towards Decision Support in Reciprocation", includes the non-synchronous case and the proofs and appears at `http://arxiv.org/abs/1601.07965`.

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